LATERAL SWAY MOTION OF VEHICLE AND TRAILER

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Abstract. In everyday life, the joint movement of a vehicle and trailer is widespread. Accordingly, when towing a trailer, due to road irregularities, potholes and the influence of side wind, a lateral swaying movement may occur. Under certain conditions, the swinging movement can increase, even ending with the overturning of a vehicle or a trailer. It is influenced by the mass of the vehicle and the trailer, as well as the initial conditions for the movement. This article discusses how to prevent such unwanted motion modes with active motion control. Movement on a horizontal road of the vehicle and trailer mutually connected with a non-deformable coupling is studied. The case is analysed when the towing vehicle moves without slipping, but the trailer can start to slide sideways. Using the technique of classical mechanics, the differential equations of the system motion were drawn up, in which a combined model of sliding and rolling resistance was used. The equations also include adaptive control of sway damping. One of the adaptive sway damping models is created as a moment of internal forces in the coupling joint. This torque control is programmed as a function with feedback from the trailer turning angle and angular velocity. The turning on of the control moment takes place against the direction of the angular velocity, observing the turning off in the range of relatively small turning angles. The second sway damping control model was created as a supplement to the towing car drive power. Also in this model, the feedback reacts to high values of the trailer turning angle and angular velocity. The motions of the vehicle and the trailer are analysed by computer simulation, in which the sway motion is induced by crosswind interaction. For all studied cases, computer simulation results are presented graphically in the form of time histories and phase diagrams. In general, the results obtained can be used in some other similar dynamic applications.

Keywords: vehicle, trailer vibrations, sway motion, active control.

Introduction

Modern science has carried out fundamental research on how, for example, the swinging of a trailer affects the driving stability of a tractor [1]. Accordingly, these studies are based on simulation calculations performed in multibody software. The results presented in this paper are achieved for a specific combination of vehicle and trailer parameters and motion start initial conditions. However, the conclusions of this work can be extended to a wider combination of parameters that would correspond to other driving situations. For example, on the synthesis of trailer sway damping systems, as well as on the optimization of the tractor's adaptive control forces in case of sway. Important studies on limiting sway movement with the control of the front wheels of the car were carried out in work [2]. This paper presents various real-time simulation results of the nonlinear model system control using the magnitude of the coupling angle as an argument for the control of the torque vector function. This control is implemented by the electric drive of the front wheels of the car, which pulls the trailer. Studies in a five-degree-of-freedom nonlinear system in sway movements, which describe lateral, angular and rolling movements taking into account the characteristics of tires and shock absorbers, are given in work [3]. In this system, static equilibrium points are found and movement near these points is analysed. Modelling of car-like vehicles has been an active research field during the past decades. For passenger cars, kinematic models are often used in the literature for automatic steering applications [4; 5]. Corresponding models for tractor-trailer vehicles are extensively covered in [6], where a method is presented for deriving models of the general n-trailer. Kinematic models of articulated vehicles have been used successfully for both planning and control applications [7; 8]. Substantial research has been done in dynamic modelling of tractor-trailer combinations at various complexity [9-11] and a comparative evaluation between lateral dynamic models for tractor-trailer systems is done in [12; 13]. In [14], kinematic and dynamic models are compared for control design using a model predictive control approach. A comparative study of linear quadratic control and model predictive control for path-following applications is presented in [15]. Low speed path-following of a tractor-trailer system using a model predictive control approach is described in [16]. For high-speed applications the focus of tractor-trailer vehicles has been mostly on stability control [17; 18], although extensive work has been done on lateral control as well [19; 20].

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From the studies of car sway movement models, it can be concluded that the object can be described by motion equations similar to a normal pendulum or a double pendulum, which should be used in crane control tasks, for example [21]. The main difference in car sway motion tasks is that instead of the gravitational interaction component in cranes, here the motion instability can be induced or prevented by the lateral or longitudinal motion dynamics of the towing vehicle.

The basis for such study is also this paper, in which the differential equations of motion of a mechatronic system are described by the d'Alembert principle [22]. The research aims to analyze the passive or active control of the system if the sway exceeds the established angular limits of the trailer.

Equations of motion for a two degree of freedom model of sway system

The simplified vehicle model is shown in Fig. 1. The coordinate x_A of the rectilinear motion of the vehicle and the angle φ of the trailer rotation are taken as generalized coordinates in this mechanical system.

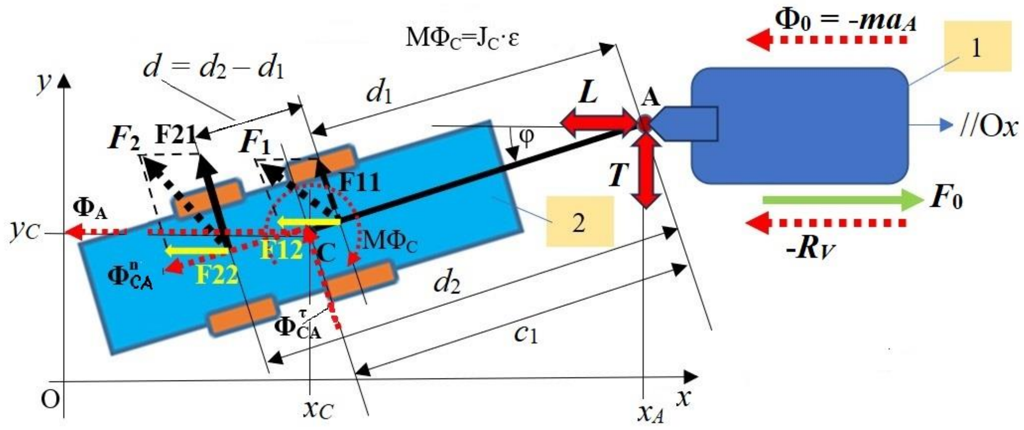

Fig. 1. **Simplified two degree of freedom vehicle model:** 1 – trailer; 2 – truck or car

In addition, it is assumed that the vehicle moves along a horizontal road. The given mechanical system is strongly non-linear because the road friction forces and wind interaction forces, acting on it, depend on the velocity of translational motion of both bodies as well as on the angular position and angular velocity of the trailer. Therefore, for such a strongly nonlinear system, the differential equations of motion can be conveniently obtained using d'Alembert's principle [22]. For this purpose, the main vector and the main moment of reduced inertial forces ($\boldsymbol{\Phi}_A$, $\boldsymbol{\Phi}^n_{CA}$, $\boldsymbol{\Phi}^\tau_{CA}$, $M\boldsymbol{\Phi}_C$, $\boldsymbol{\Phi}_0$) should be added to the coupling reactions (F_1, F_2, L, T) and the active external forces (F_0, R_V) applied to each solid body (Fig. 1).

The expressions of inertial forces and active external forces in the case considered are as follows:

$$\boldsymbol{\Phi}_A = ma_A;\ \boldsymbol{\Phi}^n_{CA} = m\omega^2 c;\ \boldsymbol{\Phi}^\tau_{CA} = m\varepsilon c;\ M\boldsymbol{\Phi}_C = J_C\varepsilon;\ \boldsymbol{\Phi}_0 = m_0 a_A; \tag{1}$$

$$F_1 = fN_1;\ F_2 = fN_2;\ R_V = -Dv_A^2 - f_0 m_0 g, \tag{2}$$

where m – mass of the trailer, kg;
J_C – trailer moment of inertia at the centre C of mass, kg·m^2;
m_0 – mass of the car, kg;
a_A – tangential acceleration of the car, m·s^{-2};
ω – angular velocity of the trailer, rad·s^{-1};
ε – angular acceleration of the trailer, rad·s^{-2};
φ – turning angle of the trailer relative to the longitudinal axis of the truck, rad;
c – distance between the trailer centre C of mass and the coupling point A, m;
F_1, F_2 – frictional forces of front and rear wheels of the trailer in interaction with road, N;
N_1, N_2 – normal reactions of front and rear wheels of the trailer in interaction with road, N;
f – rolling and sideslip reduced friction coefficient;
R_V – combined force of rolling friction and air resistance of the car, N;
D – proportionality coefficient, N·m^{-2}·s^2;

v_A – velocity of the car, m·s^{-1};
f_0 – coefficient of frictional and rolling resistance of the car;
g – acceleration of free fall, m·s^{-2}.

It should be noted that directions of forces F_1 and F_2 are dependent on the direction of the absolute velocity of the corresponding contact point, which can be written vectorially in the xOy plane as follows:

$$F_1 = \begin{pmatrix} F_{1x} \\ F_{1y} \end{pmatrix} = -kmgf \cdot \begin{pmatrix} 1 + \dfrac{\omega d_1}{\upsilon_A} \cdot \sin\varphi \cdot \mathrm{sign}\omega \\ 0 + \dfrac{\omega d_1}{\upsilon_A} \cdot \cos\varphi \cdot \mathrm{sign}\omega \end{pmatrix}, \tag{3}$$

$$F_2 = \begin{pmatrix} F_{2x} \\ F_{2y} \end{pmatrix} = -(1-k)mgf \cdot \begin{pmatrix} 1 + \dfrac{\omega d_2}{\upsilon_A} \cdot \sin\varphi \cdot \mathrm{sign}\omega \\ 0 + \dfrac{\omega d_2}{\upsilon_A} \cdot \cos\varphi \cdot \mathrm{sign}\omega \end{pmatrix}, \tag{4}$$

where k – determines the trailer weight mg distribution between the wheels ($0 \le k \le 0{,}5$, i.e. if $k = 0$ – the entire load falls on one wheel, and if $k = 0.5$ – the load is distributed equally between two wheels);
signω – +1 or -1, depending on whether $\omega > 0$ or $\omega < 0$;
d_1, d_2 – distances of the front and rear wheels to the coupling point A (Fig. 1).

To compile differential equations of motion of a vehicle system, the kinetostatics method is used [23]. The equation of kinetostatic equilibrium in projection onto the x axis has the following form:

$$-(m + m_0)a_A - m\omega^2 c \cdot \cos\varphi - m\varepsilon c \cdot \sin\varphi + F_{1x} + F_{2x} + F_0 - D\upsilon_A^2 - f_0 m_0 g = 0. \tag{5}$$

Accordingly, the kinetostatic equilibrium equation of the trailer for the sum of moments of forces relative to the coupling point A has the following form:

$$-J_C\varepsilon - mc^2\varepsilon - ma_A c \cdot \sin\varphi + M(F_1) + M(F_2) + M_{ad} = 0. \tag{6}$$

Expressions for moments $M(F_1)$ and $M(F_2)$ in equation (6) are as follows:

$$M(F_1) = F_{1x} \cdot d_1 \cdot \sin\varphi - F_{1y} \cdot d_1 \cdot \cos\varphi; \tag{7}$$

$$M(F_2) = F_{2x} \cdot d_2 \cdot \sin\varphi - F_{2y} \cdot d_2 \cdot \cos\varphi, \tag{8}$$

where $M(F_1)$ and $M(F_2)$ – moments of forces F_1 and F_2 against point A;
F_0 – traction force produced by the wheels of the car;
M_{ad} – additional moment (for example, the moment from the interaction in the coupling pivot, moment due to the effect of crosswind, moment from the active or passive oscillation damping mechanism in the coupling).

Taking into account that $a_A = \frac{d^2x}{dt^2}$ and $\varepsilon = \frac{d^2\varphi}{dt^2}$, equations (5) and (6) can be transformed to the following form:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = f_1(\upsilon_A, \varphi, \omega) \text{ and } \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = f_2(\upsilon_A, \varphi, \omega), \tag{9}$$

where $f_1(\upsilon_A, \varphi, \omega)$; $f_2(\upsilon_A, \varphi, \omega)$ – analytical relations obtained from equations (5) and (6) for the unknown a_A and ε.

Numerical modelling of equations (9) under the given initial conditions (time $t = 0$, $x_A(0)$, $v_A(0)$, $\varphi(0)$, $\omega(0)$) allows solving analysis and synthesis problems for motion control. For example, by solving of analysis task, the coupling reactions L and T can be found (Fig. 1). Then, the non-slip of the rear wheels of the towing car can be checked or prevented by changing the system parameters. Accordingly, in the synthesis problems, it is possible to analyse the adaptive control of sway elimination by changing the force F_0 or the interaction moment M_{ad} (see equations (5)-(6)). Some examples of solving such problems are given below.

Description of crosswind interaction with a trailer and damping of sway motion

Two types of functions were used to describe the interaction of side wind gust with a trailer: (a) wind gust as the harmonic pulse; (b) wind gust as the rectangular impulse. In these cases, mathematical descriptions of additional components of moment M_{ad} are as follows:

$$M1w_{ad} = Q_0 c \cdot \cos\varphi \cdot \sin(pt) \cdot (0.5 + 0.5 \cdot \text{sign}(\pi - pt); \tag{10}$$

$$M2w_{ad} = Q_0 c \cdot \cos\varphi \cdot (0.5 + 0.5 \cdot \text{sign}(\sin(pt)) \cdot (0.5 + 0.5 \cdot \text{sign}(\pi - pt)), \tag{11}$$

where $M1w_{ad}$, $M2w_{ad}$ – additional components of function M_{ad} in equation (6);
Q_0 – crosswind force at the mass centre C of the trailer, at a distance c;
p – constant.

Accordingly, during the numerical modelling, two control functions were chosen for damping the sway movement: (a) dry friction interaction damping adaptive moment $M3_{ad}$ at the coupling point A; (b) adaptive traction control force $F_0 = F_{ad}$ of the vehicle drive. Mathematical descriptions of $M3_{ad}$ and F_{ad} are as follows:

$$M3_{ad} = M_0 \cdot \text{sign}\omega \cdot \left(0.5 - 0.5 \cdot \text{sign}\left(\frac{\pi}{q} - / \varphi /\right)\right) \quad ; \tag{12}$$

$$F_0 = F_{ad} = \left(1 + S \cdot \text{sign}(\omega\varphi)\right) \cdot \left[D \cdot (v_A(0))^2 + f_0 m_0 g + fmg\right], \tag{13}$$

where M_0 – module of control moment;
q – constant;
/φ/ – absolute value of the trailer turning angle φ;
S – small constant;
$v_A(0)$ – velocity of the vehicle in stationary motion without sway;
other notations are the same as in equations (3)-(8).

The proposed crosswind interaction relations (10)-(11) and effects of adaptive control of trailer sway damping (12)-(13) can be used in modelling and parametric synthesis of real systems. Two such motion cases are discussed in the next section.

Numerical simulation results and discussion

The results of sway damping simulation obtained using the developed methodology are presented in Fig. 2-9. The first example of numerical simulation (Fig. 2-5) refers to the action on the system of a single harmonic crosswind pulse in accordance with equation (10). Besides, adaptive control of the trailer sway damping in this case is made by the vehicle traction force variation according to (13).

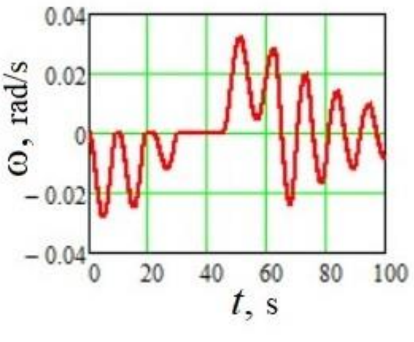

Fig. 2. **Change in the angular velocity *ω* of the trailer in time *t***

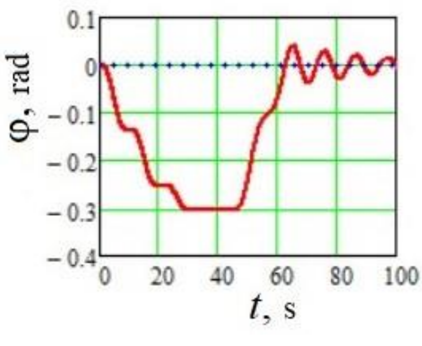

Fig. 3. **Change in the turning angle *φ* of the trailer in time *t***

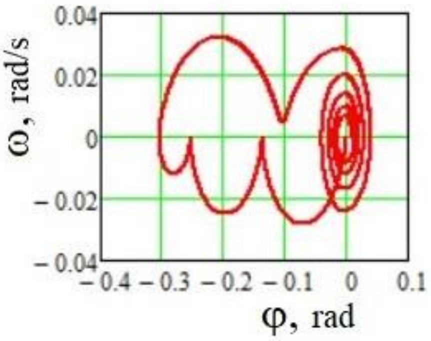

Fig. 4. **Representation of the trailer motion on the phase plane (*φ*, *ω*)**

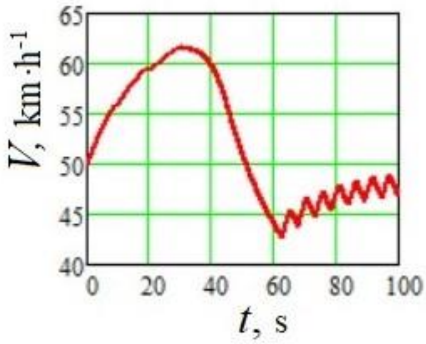

Fig. 5. **Change in the speed *v* of the towing vehicle in time *t***

The simulation results in Fig. 2-5 are obtained under the following system parameters: m = 500 kg; Jc = 1208 kg·m^2; m_0 = 2000 kg; k = 0.5; $d_1 = d_2 = c$ = 2 m; f = 0,2; f_0 = 0.05; D = 5 kg·s^{-1}; S = 0.2; g = 9.81 m·s^{-2}; Q_0 = 500 kg·m·s^{-2}; p = 0.05 rad·s^{-1}; $v_A(0) = 50\cdot(3.6)^{-1}$ = 13.89 m·s^{-1}.

As follows from the modeling results, under the influence of side wind gusts, intense swaying of the trailer occurs; due to this, the turning angle φ of the trailer gradually increases and at t = 28 s reaches 0.3 rad (Fig. 3). To eliminate such angular drifts of the trailer, adaptive control of the vehicle traction force F_0 is used. As the result, the speed v of the towing vehicle is increased initially from 50 till 62 km·h^{-1} (Fig. 5), and this causes a gradual decrease of the trailer turning angle to $\varphi \approx 0$ (Fig. 3). But the speed of the towing vehicle returns back to the its initial value of 50 km·h^{-1} (Fig. 5).

Another adaptive control action is considered in the second modelling example (Fig. 6-9). In this case, the rectangular impulse $M2w_{ad}$ of the cross-wind gust according to (11) was applied, while damping of the sway movement $M3_{ad}$ was performed with the control of the coupling adaptive torque (12). The main parameters of the system are taken as follows: M_0 = 750 N·m; q = 20; $v_A(0)$ = 19.4 m·s^{-1}.

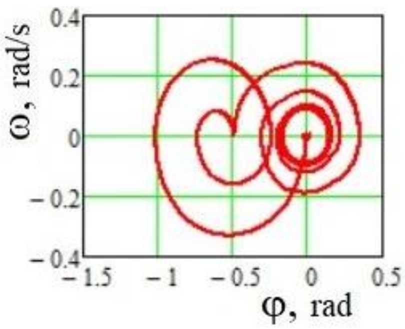

Fig. 6. **Representation of the trailer motion on the phase plane (*φ*, *ω*)**

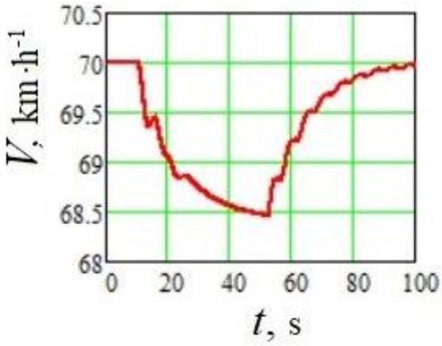

Fig. 7. **Change in the speed *v* of the towing vehicle in time *t***

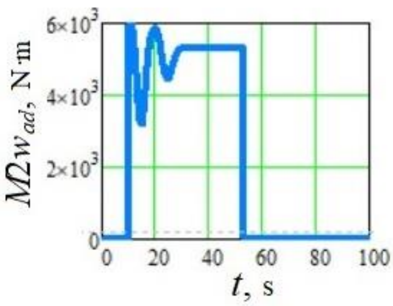

Fig. 8. **Moment $M2w_{ad}$ of the rectangular impulse of cross-wind gust**

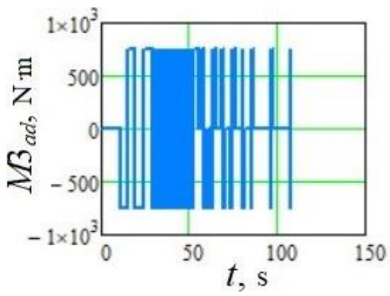

Fig. 9. **Change in time *t* of the adaptive damping moment $M3_{ad}$ at the coupling point**

The modelling results show that the developed sway motion analysis method is very effective. It allows testing different variants of damping of sway oscillations by studying a two-degree-of-freedom pendulum-type motion system. Accordingly, two options are proposed for adaptive damping control. These damping systems have very simple feedback. They are described only by constant level functions of the phase coordinates φ, ω (i.e. sign = sign $(\varphi, \omega) = \pm 1$), without the need to calibrate the sensor over the entire range of coordinate changes. When evaluating the described method of sway analysis, it should be noted that it continues with determination of the reaction forces L and T (Fig. 1). To do this, one should apply the kinetostatic equations for the trailer in projections on the Ox and Oy axes as follows:

$$L - F_{1x} - F_{2x} - m\omega^2 c \cdot \cos\varphi - m\varepsilon c \cdot \sin\varphi \;=\; 0, \tag{14}$$

$$T \;+\; F_{1y} \;+\; F_{2y} - m\omega^2 c \cdot \sin\varphi \;+\; m\varepsilon c \cdot \cos\varphi \;=\; 0. \tag{15}$$

The reactions L and T, found from equations (14)-(15), can be used to check the non-slip of the rear wheels of the towing vehicle. During this test, the kinetostatic method for the towing vehicle should additionally be used, taking into account the friction forces in the front and rear wheels. This relatively simple question is not considered here due to space limitations.

Conclusions

1. In this paper, an analytical methodology is developed for the analysis of two-degree-of-freedom sway motion and adaptive damping control of a vehicle.
2. To suppress sway movement, two types of adaptive control have been proposed: (a) control as an additional change in the moment of internal forces at the coupling point; (b) control as an additional component of the driving force of the towing vehicle.
3. The methodology allows analysis of sway motion damping using numerical simulations when interacting with crosswinds.
4. The principle of the proposed method can also be used to analyse the sway motion of other vehicles with more than two degrees of freedom.
5. The proposed method makes it possible to analyze the sway of a vehicle on a slippery surface (icy highway), using a small sideslip reduced friction coefficient f, approximately $f < 0.05$.
6. The numerical values from the computer experiments of the present study were used to determine the parameters of a real traffic accident.

Author contributions

Conceptualization, J.V.; methodology, V.B. and E.K.; software, M.I. and M.E.; validation, V.B. and M.I.; formal analysis, J.V. and E.K..; investigation, V.B., M.E. and J.V.; writing – original draft preparation, J.V. and V.B.; writing – review and editing, V.B.; visualization, V.B.; project administration, V.B.; funding acquisition, J.V. All authors have read and agreed to the published version of the manuscript.

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